A GENERALIZED FORMULA FOR THE TRANSFORM OF A PRODUCT OF FUNCTIONS

(OBOBSHCHENNAIA FORMULA DLIA IZOBRAZHENIIA PROIZVEDENIIA ORIGINALOV)

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A generalization will be given of a formula of Natanzon's [1] for the transform of a product of functions to the particular case when one of the functions in the product is complicated. Let

$$\Phi(p) = \int_0^\infty F[q(t)]\,\psi(t)\,e^{-pt}dt \qquad (1)$$

On the basis of a known theorem, thanks to Efros, we have

$$F[q(t)]\,\psi(t) = \int_0^\infty e^{-tu}du \int_0^\infty f(v)\,g(u,v)\,dv \qquad (2)$$

where

$$F(p) = f(t), \qquad \psi(p)\,e^{-vq(p)} = g(t,v)$$

Substituting (2) into equation (1), we obtain

$$\Phi(p) = \int_0^\infty\int_0^\infty\int_0^\infty f(v)\,g(u,v)\,e^{-t(p+u)}\,dt\,du\,dv$$

Interchanging the order of integration in this formula and noting that

$$\int_0^\infty g(u,v)\,e^{-tu}du = \psi(t)\,e^{-vq(t)}$$

we get

$$\Phi(p) = \int_0^\infty\int_0^\infty f(v)\,\psi(t)\,e^{-vq(t)-pt}\,dv\,dt$$

Putting $G(p, v) = \psi(t)e^{-vq(t)}$, we finally arrive at the formula

$$\Phi(p) = \int_0^\infty f(v)\,G(p, v)\,dv \qquad (3)$$

As a particular case, we may obtain from (3) the formula for the transform of a product of functions. In fact, putting $q(t) = t$, we have $G(p, v) = \Psi(p + v)$, where $\Psi(p) = \psi(t)$, and (3) gives

$$\Phi(p) = \int_0^\infty F(t)\,\Psi(t)\,e^{-pt}dt = \int_0^\infty f(v)\,\Psi(p+v)\,dv = \int_p^\infty f(v-p)\,\Psi(v)\,dv \qquad (4)$$

An application to the transformation of Carson-Heaviside's formula, similar to formula (4), has been given by Natanzon [1].

It is likewise easy to deduce from (3) a formula for the transform of the quotient of two functions. In this case, one has $F(p) = 1/p$ and $f(t) = 1$. Then

$$\Phi(p) = \int_0^\infty \frac{\psi(t)}{q(t)}\,e^{-pt}dt = \int_0^\infty G(p, v)\,dv \qquad (5)$$

In a similar manner, we may obtain other particular versions of the general formula (3).

BIBLIOGRAPHY

1. Natanzon, V.Ia., Formula dlya izobrazheniya proizvedeniya originalov (A formula for the transform of a product of functions). *PMM* Vol. 20, No. 5, 1956.

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